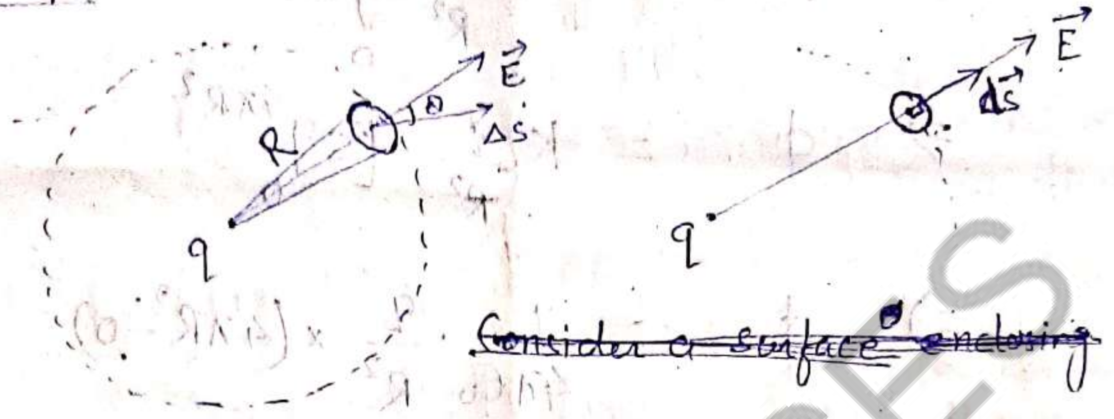
Lecture - 10

Gauss. Law :→/Theorem :- The total Electric flux through a closed surface enclosing charge is $\left(\frac{q}{\epsilon_0}\right)$.

Proof :-



~~Consider a surface enclosing~~

Consider a charge "q" enclosed by a surface and electric field $\vec{E}$ passing through the surface is $\vec{ds}$. and flux ejected from this area is $d\phi$.

Now according to diff of flux! -

$$\Rightarrow \quad d\phi = \vec{E}\cdot\vec{ds}$$

$$\Rightarrow \quad d\phi = E\,ds\cos\theta$$ { If vector surface area in the direction of field then $\theta = 0$

$$\Rightarrow \quad d\phi = E\,ds\cos 0$$

$$\Rightarrow \quad d\phi = E\,ds$$

for total flux Integrating both side we get

$$\Rightarrow \quad \int d\phi = \int E\,ds$$

ϕ tends from 0 to ϕ

then S tends from 0 to $4\pi R^2$ (Surface are of Sphere) According to fig.

$$\Rightarrow \quad \int_0^{\phi} d\phi = \int_0^{4\pi r^2} E\,ds$$

$$\Rightarrow [\phi]_0^{\phi} = \int_0^{4\pi R^2} K\frac{q}{R^2}\, ds \quad \left\{ \because E = K\frac{q}{R^2} \right\}$$

$$\Rightarrow \phi - 0 = K\frac{q}{R^2} \int_0^{4\pi R^2} ds$$

$$\Rightarrow \phi = K\frac{q}{R^2} [S]_0^{4\pi R^2}$$

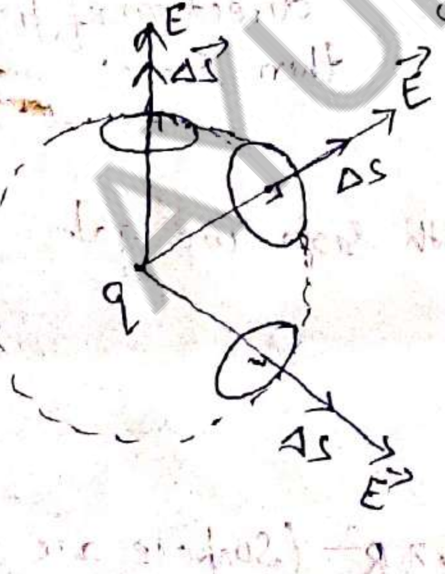
$$\Rightarrow \phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \times (4\pi R^2 - 0)$$

$$\Rightarrow \phi = \frac{1}{\cancel{4\pi}\epsilon_0} \cdot \frac{q}{\cancel{R^2}} \times \cancel{4\pi R^2}$$

$$\Rightarrow \boxed{\phi = \frac{q}{\epsilon_0}}$$

Method-II (Alternative Method) :-

According to diff of Flux



$$\Delta\phi = \vec{E} \cdot \vec{\Delta S}$$

If all surface of spherical part ~~def~~ devided into small surface of surface area $\vec{\Delta S}$ through which Electric field crossing through the surface $\vec{E}$. Then total flux is equal to sum of flux ejected through this area.

$$\Phi_{Total} = \sum \Delta\phi$$

$$\Phi_{total} = \sum \vec{E} \cdot \vec{\Delta S}$$

$$= \cancel{\sum \frac{Kq}{4\pi\epsilon}}$$

$$\Phi_{total} = \sum E \Delta S \cos 0 \quad \left\{ \vec{E} \,\&\, \vec{\Delta S} \text{ is in the Same direction} \right.$$

$$\Phi_{total} = \sum E \Delta S$$

$$\Phi_{total} = E \sum \Delta S \quad \left\{ \text{If } E \text{ is Constant for all surface A/fig} \right\}$$

$$\Phi_{total} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \left(\sum \Delta S \right)$$

$$\left\{ \text{Here } \sum \Delta S = \text{Surface area of Sphere} = 4\pi R^2 \right\}$$

$$\Phi_{total} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \times 4\pi R^2$$

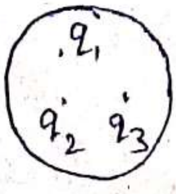
$$\boxed{\Phi_{total} = \frac{q}{\epsilon_0}}$$

Proved

Significance of Gauss' Law:-

→ It is true for any close surface no matter what its shape. But not easy to apply every shape.

→ Charge / system of Charge may be located any where inside the Surface.



$$\boxed{\Phi_{total} = \frac{q_{net}}{\epsilon_0} = \frac{q_1 + q_2 + q_3}{\epsilon_0}}$$

→ The Surface the we ~~close~~ Select for the application of Gauss law is Called Gaussian Surface.

But Gaussian surface Can pass through a Continuous Charge distribution.

→ It is useful towards a much easier Calculation of Field when system has Some Symmetry.

* Application of Gauss law :→ It is applicable for obtain electric Field due to Symmetric Charge distribution in simple way.

For this we assume a Gaussian surface where we have to find field.

In Syllabus :-

(A) Field Due to an Infinitely long straight Uniformly Charge wire.

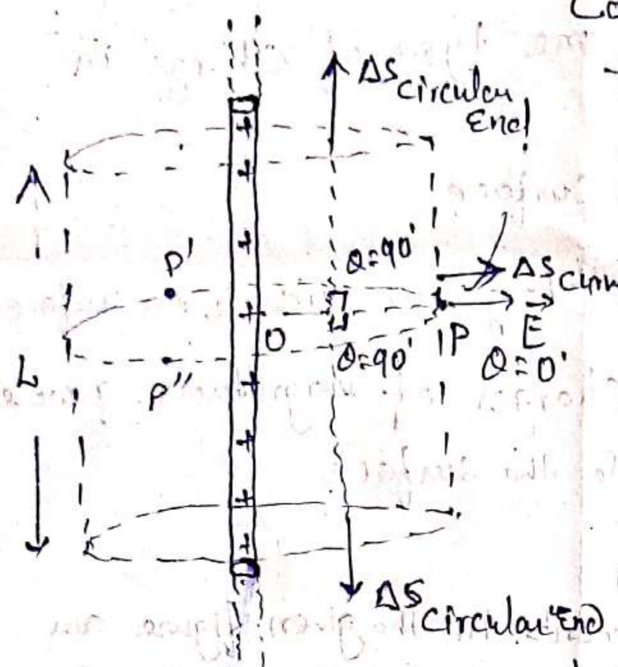
(B) Field due to a Uniformly Charged Infinite plane sheet

(C) Field due to a Uniformly charged thin Spherical Shell

(D) Field due to Uniformly Charged Sphere.

Application of Gauss law

1.) Field due to an infinitely long straight Uniformly Charged wire :→



Consider an infinitely long thin straight wire of Charge density "λ". This wire is at axis of Symmetry Let $\vec{OP} = \vec{r}$ is radial Vector rotate it arround wire and the point obtain P, P', P'' Completely equivalent with rest to charged wire.

To Calculate the field we imagine Cylindrical Gaussian Surface.

Then total flux outward is

$$\Rightarrow \phi = \phi_{\text{At Curved Surface}} + \phi_{\text{At two Circular End.}}$$

$$\Rightarrow \phi = \vec{E} \cdot \vec{\Delta S}_{\text{Curved}} + \vec{E} \cdot \vec{\Delta S}_{\text{At Both Circular End}}$$

$$\Rightarrow \phi = |\vec{E}|\,|\vec{\Delta S}|\cos 0° + |\vec{E}|\,|\vec{\Delta S}| \cdot \cos 90°$$

$$\Rightarrow \phi = E\,\Delta S + 0$$

$\left\{ \text{from Gauss law } \phi = q/\epsilon_0 \right\}$

$$\Rightarrow \frac{q}{\epsilon_0} = E\,\Delta S_{\text{curved}}$$

$$\Rightarrow \frac{q}{\epsilon_0} = E\,(2\pi r L)$$

$$\cancel{E = \frac{q}{2\pi r L}}$$

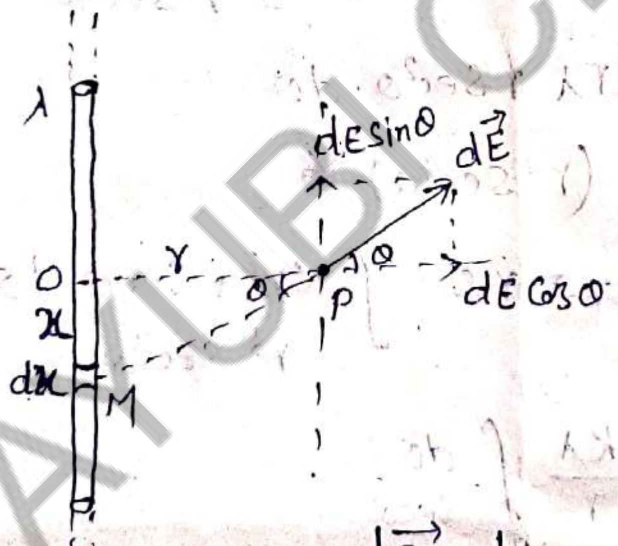
$$E = \frac{q}{2\pi r L \epsilon_0}$$

$$E = \frac{q/L}{2\pi \epsilon_0 \cdot r}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \{ q/L = \lambda \text{ Line Charge density} \}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \cdot \hat{r}} \quad \{ \text{direction radialy}$$

N.C.E.R.T Q. 1.30 Obtain Electric field due to a long thin wire of Uniform Linear Charge density E without using Gauss law.



let for long thin wire. Small elementry length "dx" field at p is "$d\vec{E}$" at "θ" angle The position of "dx" from "O" is at "x" distance

$$\left|\vec{E}_{net}\right| = \int dE\cos\theta \quad — ①$$

because net Electric field only in radial direction so only cos component of field, sine Component Cancle due to resuting both end of wire.

$$MP^2 = \left(\sqrt{OM^2+OP^2}\right)^2 = \left(\sqrt{x^2+r^2}\right)^2 = x^2+r^2$$

$$\cos\theta = \frac{r}{OM} = \frac{r}{\sqrt{x^2+r^2}}$$

$$dE\cos\theta = \frac{K\,dQ}{MP^2}\cos\theta = \frac{K\,\lambda\,dx}{x^2+r^2} \times \frac{r}{\sqrt{x^2+r^2}}$$

$$dE\cos\theta = \frac{K r \lambda\, dx}{(x^2+r^2)^{3/2}}$$

from Eqⁿ ①

$$|\vec{E}_{net}| = \int \frac{K r \lambda\, dx}{(x^2+r^2)^{3/2}} \quad \text{— (11)}$$

Now let $\tan\theta = \frac{x}{r} \Rightarrow x = r\tan\theta$

$$dx = r\sec^2\theta \cdot d\theta$$

$$x^2+r^2 = r^2\tan^2\theta + r^2 = r^2(\tan^2\theta+1)$$
$$= r^2\sec^2\theta$$

$$E_{net} = \int \frac{K r\lambda\, r\sec^2\theta \cdot d\theta}{(r^2\sec^2\theta)^{3/2}}$$

$$E_{net} = \cancel{Kr\lambda r s} \int \frac{K r^2 \lambda \sec^2\theta\, d\theta}{r^3\sec^3\theta}$$

$$E_{net} = \frac{K\lambda}{r}\int \frac{d\theta}{\sec\theta}$$

$$E_{net} = \frac{K\lambda}{r}\int_{\alpha}^{\beta}\cos\theta\, d\theta \quad \{\text{take randomly limit } \alpha \text{ to } \beta\}$$

$$E_{net} = \frac{K\lambda}{r}\left[\sin\theta\right]_{\alpha}^{\beta}$$

$$\boxed{E_{net} = \frac{K\lambda}{r}(\sin\beta - \sin\alpha)}$$

For Infinite wire

$\alpha = -\pi/2 \quad \beta = \pi/2$

$$E_{net} = \frac{K\lambda}{r}\left[\sin \pi/2 - \sin(-\pi/2)\right]$$

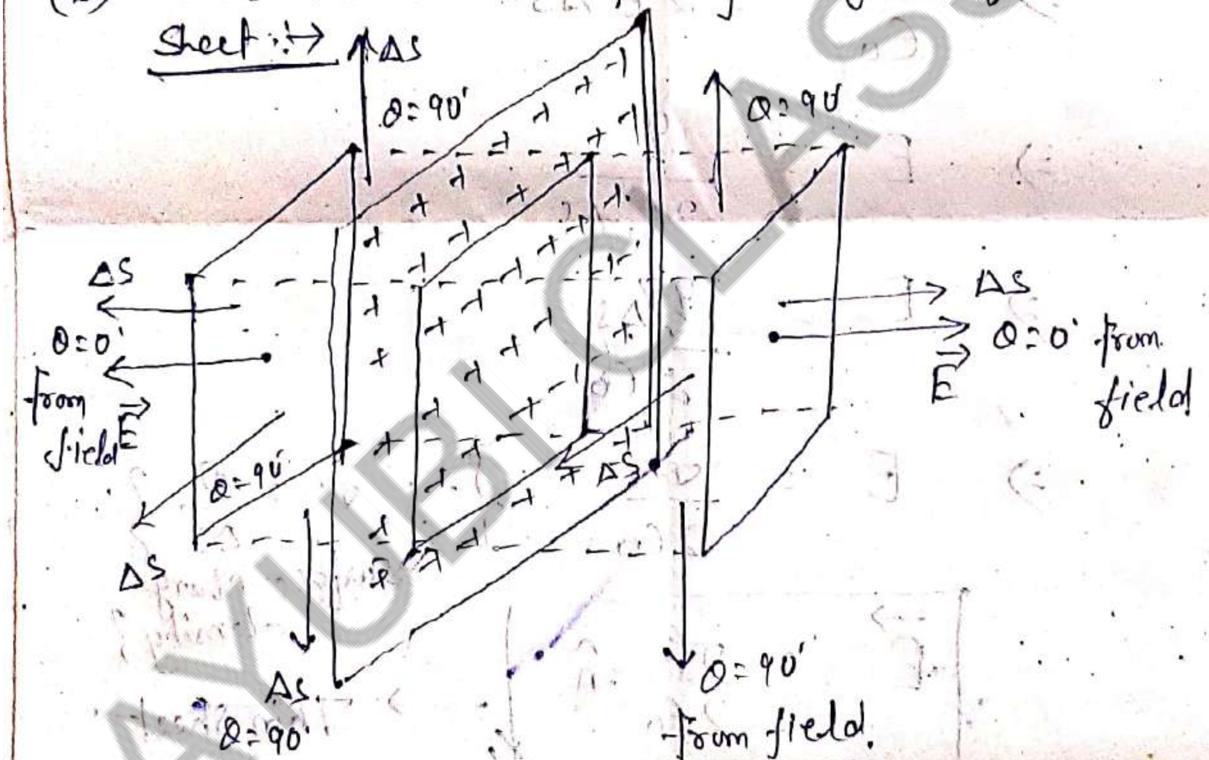
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{r} \, 2$$

$$E = \frac{\lambda}{2\pi\epsilon_0 \cdot r}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \cdot \hat{r}}$$

Lecture-13

(B) Field due to a Uniformly Charged infinite plane Sheet :-



Let "σ" be the uniform surface charge density of an infinite plane sheet.

We take the x-axis normal to the given plane and according to symmetry field will not y & z co-ordinate and its direction but it will parallel to x-axis only at every point.

So, Total flux only in x - direction both side of sheet.

$$\Rightarrow \Phi_{Total} = 2\Phi$$

$$\Rightarrow \Phi_{Total} = 2\vec{E}\,\Delta\vec{S}$$

$$\Rightarrow \Phi_{Total} = 2E\Delta S \cos 0°$$

$$\Rightarrow \Phi_{Total} = 2E\Delta S$$

$$\left\{ \text{Gauss law } \Phi_{Total} = q/\epsilon_0 \right\}$$

$$\Rightarrow \frac{q}{\epsilon_0} = 2E\Delta S$$

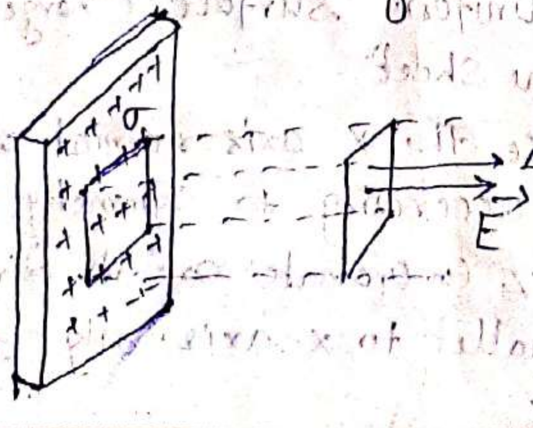
$$\Rightarrow E = \frac{q}{2\epsilon_0 \Delta S}$$

$$\Rightarrow E = \frac{q/\Delta S}{2\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0} \quad \left\{ \sigma = \frac{Q}{\Delta S} \text{ surface charge density} \right\}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\,\hat{n}} \longrightarrow \text{Thin sheet}$$

Case - II In Case of Thick sheet.



In this condition ~~sheet~~ only one surface will affect the surface in particular direction normal to that surface.

So.

$$\phi_{Total} = \vec{E}\cdot\vec{\Delta S}$$

$$\phi_{Total} = E \Delta S \cos 0^{\circ}$$

$$\frac{q}{\epsilon_0} = E \Delta S$$

$$E = \frac{q/\Delta S}{\epsilon_0}$$

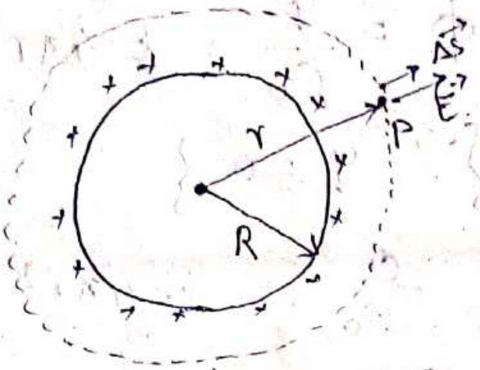
$$E = \frac{\sigma}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}} \rightarrow \text{Thick sheet.}$$

Lect - 14.

(C) Field due to Uniformly Charged Spherical shell.

(i) Field outside the shell:–



Consider a point "P" out side the shell with radious vector. r. To Calculate $\vec{E}$ at P. we assume Gaussian surface of radious "r." If radious of shell is "R".

According to diff of Flux

$$\phi = \vec{E} \cdot \vec{\Delta S}$$

$$\phi = E \Delta S \cos 0$$

$$\phi = E\, 4\pi r^2 \quad \text{---} (1) \qquad \{\Delta S = 4\pi r^2$$

$$\downarrow$$

from Gauss law $\dfrac{q}{\epsilon_0} = E \cdot 4\pi r^2$

$$\therefore E = \frac{q}{4\pi \epsilon_0 r^2}$$

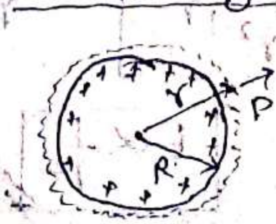
$$\boxed{\vec{E} = \frac{q}{4\pi \epsilon_0 r^2} \cdot \hat{r}} \qquad r \geq R \qquad \boxed{E \propto \frac{1}{r^2}}$$

This, however, is exactly the field produce by a charge "q" placed at centre. "O".

Thus for point outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre.

(ii) Field very close to surface!–

$r \approx R$



$$\boxed{E = \frac{q}{4\pi \epsilon_0 R^2} \cdot \hat{r}} \qquad r = R$$

charge enclosed by Gaussian surface is zero.

$$\phi = \frac{q}{\epsilon_0} = E \cdot 4\pi r^2$$

$$\{\because q = 0\}$$

$$\Rightarrow \frac{0}{\epsilon_0} = E \cdot 4\pi r^2$$

$$\Rightarrow \boxed{E = 0} \quad r < R$$

The important result is a direct consequence of Gauss's law which follow from Coulomb's law. The experimental verification of this result conferm's the $1/r^2$ dependence in Coulomb's law.

(D) field Due to Uniformly Charged Solid Sphere:-

In Solid sphere charge exist in all parts of sphere (entire volume). Then field calculation at out side and very close to sphere is smilar as spherical shell but inside the sphere is depend on charge ~~dis~~ distribution accross volume. (Volume charge Density ρ)

Now:- (i) Point out side sphere:-

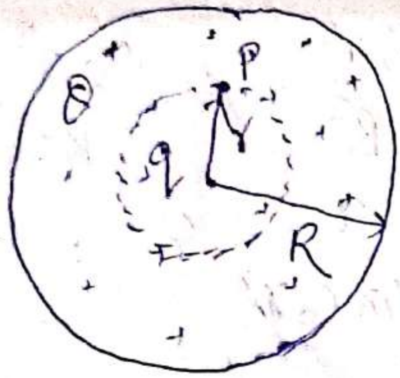
$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}} \quad r > R$$

$$\boxed{E \propto \frac{1}{r^2}}$$

(ii) Point very close to sphere

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \hat{r}} \qquad r = R$$

(iii) Point Inside Sphere :-



Consider a sphere of Uniformly Charged "Q".
Let a point P inside the sphere P where Charge enclosed by Gaussian Surface of radious "r" is "q".

Volume charge density $\rho = \frac{Q}{V}$

A/fig $\quad \rho = \frac{Q}{V} = \frac{q}{V'}$

$$\Rightarrow \frac{Q}{\cancel{\frac{4}{3}\pi} R^3} = \frac{q}{\cancel{\frac{4}{3}\pi} r^3}$$

$$\Rightarrow \boxed{q = \frac{Q r^3}{R^3}}$$

Now Here this Value of q enclose by Gaussian Surface.

then $\quad \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \cdot \hat{r}$

$$\vec{E} = \frac{Q r^3}{4\pi\epsilon_0 \cdot r^2 \cdot R^3} \cdot \hat{r}$$

$$\boxed{\vec{E} = \frac{Q r}{4\pi \epsilon_0 R^3} \cdot \hat{r}}$$ $r < R$

$\boxed{E \propto r}$

(iv) <u>At the centre</u> :- $r = 0$

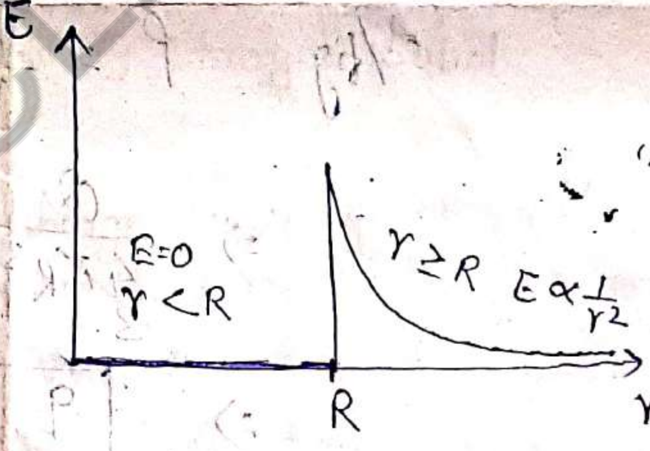
$$q = \frac{Q r^3}{R^3} = \frac{Q \cdot 0^3}{R^3} = 0$$

then $$\vec{E} = \frac{q}{4\pi \epsilon_0 r^2} \hat{r} = 0$$

$$\boxed{\vec{E} = 0}$$

× <u>Graphical Representation of field in Sphere</u>:-

(A) <u>Spherical Shell</u> :-



(B) <u>Solid sphere</u> :-